1. kolokvij iz Matematike
(Ljubljana, 2. 12. 2015)

Time limit: 90 minutes. All problems are worth the same amount of points. Read the full text of each problem carefully. You may use two A4 sheets with formulas. The results will be available at `ucilnica.fri.uni-lj.si`*.*

Justify all your answers!

1. You are given two complex numbers, $a = 1 + i$ and $b = -1 + i$.

    (a) Express the numbers a and b in the polar form, i.e. in the form $re^{i\phi}$.

    (b) Evaluate $\left(\frac{a}{b}\right)^{2015}$.

2. A sequence (a_n) is given by

$$a_n = \frac{2^{n+1}}{1 + 2^n}.$$

    (a) Evaluate the limit $\lim_{n \to \infty} a_n$.

    (b) Show that (a_n) is an increasing sequence.

3. A function f is given by $f(x) = x\sqrt{4 - x^2}$.

    (a) Determine the domain of definition D_f of the function f.

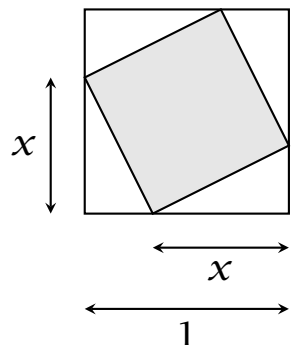   (b) Find the derivative f' of the function f.

    (c) Find the equation of the tangent line to the graph of f through the point $(\sqrt{3}, f(\sqrt{3}))$.

4. Into a square with an edge of length 1 is inscribed a smaller square such that its vertices lie on the edges of the larger square. What is the smallest possible area of the inscribed square?

    (a) Find the formula for the function $p(x)$ which gives you the area of the inscribed square.

    (b) What is the largest possible closed interval that contains x?

    (c) Find the smallest value of the function $p(x)$ on this interval.

Justify all your answers!