2. kolokvij iz Matematike
(Ljubljana, 14. 1. 2016)

Time allowed: 90 min. All of the problems are equivalent. Please read the text of each problem carefully. Two A4 sheets with formulas are allowed. Results will be at `ucilnica.fri.uni-lj.si`.

All of the answers have to be justified!

1. Functions f and g are defined as follows
$$f(x) = x^3 - 3x + 1 \text{ and } g(x) = x^2 + 3x + 1.$$
   (a) Find all the points at which the graphs of the functions intersect.
   (b) Calculate the area of the smallest region, that is enclosed by the graphs of f and g.

2. Find the volume of the solid of revolution, created by rotating the graph of the function
$$h(x) = \sqrt{x}e^{-\frac{x^2}{2}}$$
   on the interval $[0, 1]$ around the x-axis.

3. You are given points $A(1, 3, 2)$, $B(4, 0, 8)$, and $C(4, 2, 6)$.
   (a) Show that the given points do not lie on the same line.
   (b) Calculate the area of the triangle $\triangle ABC$.
   (c) Find the point D on the segment AB such that $\overrightarrow{CD} \perp \overrightarrow{AB}$.

4. You are given points $A(1, 2, 1)$, $B(1, -1, 2)$, and $C(1, 1, 3)$ and a plane
$$\Sigma : x - y + 2z = 6.$$
   (a) Find the canonical equation of the line p through A and B.
   (b) Which of the points A, B, or C lie on the plane Σ?
   (c) Find the point of intersection of the line p and the plane Σ.

All of the answers have to be justified!