1. kolokvij iz Matematike
(Ljubljana, 22. 11. 2016)

"Time limit: 90 minutes. Read the full text of each problem carefully. You may use two A4 sheets with formulas. The results will be available at `ucilnica.fri.uni-lj.si`.

$$\boxed{\textbf{Vse odgovore dobro utemelji!}}$$

1. **[25 točk]** For the complex equation

$$a + 2\bar{a} = \frac{-3i + \sqrt{3}}{i}.$$

   (a) Find the solution a.

   (b) Write a in polar coordinates and find all of the solutions for the equation $z^3 = a$.

2. **[20 točk]** Calculate the limit $\lim\limits_{n\to\infty} \sqrt{\dfrac{2n+1}{n+1}}$ and show that the sequence $a_n = \sqrt{\dfrac{2n+1}{n+1}}$ is increasing.

3. **[25 točk]** Given the infinite series $\sum_{n=1}^{\infty} \frac{2}{n^2+2n}$.

   (a) Prove that its n-th partial sum is given by the formula

$$s_n = \frac{n(3n+5)}{n(2n+6)+4}.$$

   (b) Calculate $\lim_{n\to\infty} s_n$. Is the series convergent? If yes, what is the sum?

4. **[30 točk]** Given the function $f(x) = \frac{2-x}{2x+3}$.

   (a) What is the domain of definition D_f and range Z_f of the given function. Is the function even, odd, injective, surjective, increasing, decreasing? Find zeroes poles, asymptotic lines of f and sketch its graph as accurate as possible.

   (b) Find the formula for the inverse function $f^{-1}(x)$.

   (c) Find the equation of the tangent line to the graph of f touching the point $(-1, f(-1))$.

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